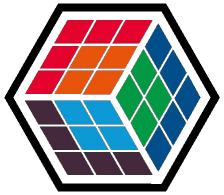


THE DEVELOPER'S CONFERENCE

Otimização de Decisões em Python

Renan Eccel

Analista de Pesquisa Operacional na WPLEX

/in/renan-eccel

/renan-eccel

renan.eccel@gmail.com

Cansado disso?

E disso?


NOVOS BOMBONS!
Garoto
GAROTICES
Serenata de Amor
CROCANTE
Garoto AO LEITE
Baton Creme
ECLIPSE
LUAR
300 g BOMBONS SORTIDOS
CACAU DO BRASIL
Garoto

E disso?


Garoto
CROCANTE


milk
&
mel

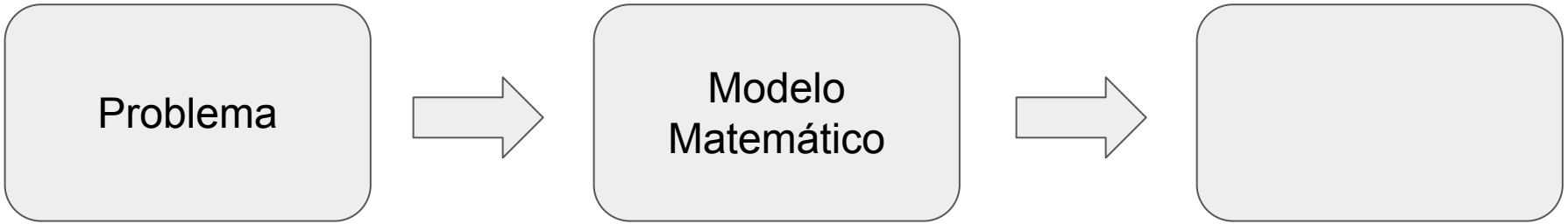

Garoto
Caribe
Bombom recheado
com banana

Seus problemas acabaram!

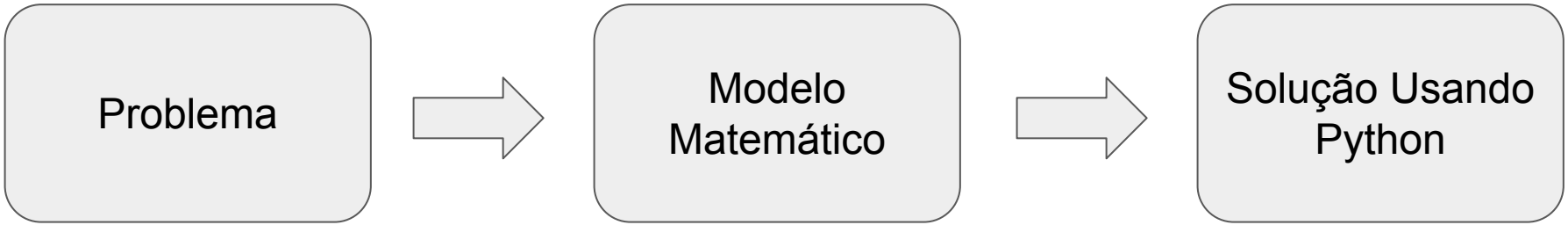
3 simples passos

3 simples passos

3 simples passos



3 simples passos



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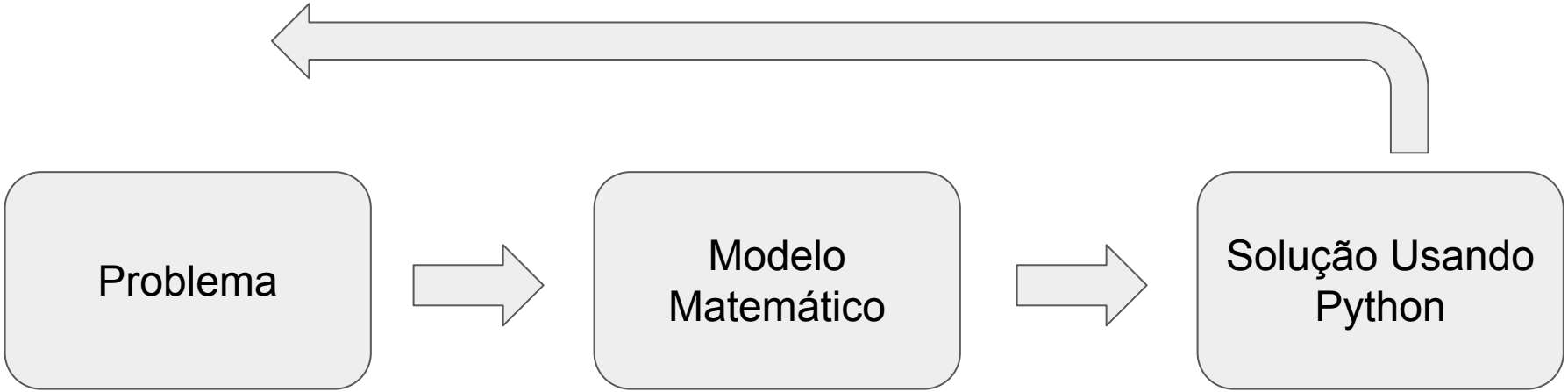

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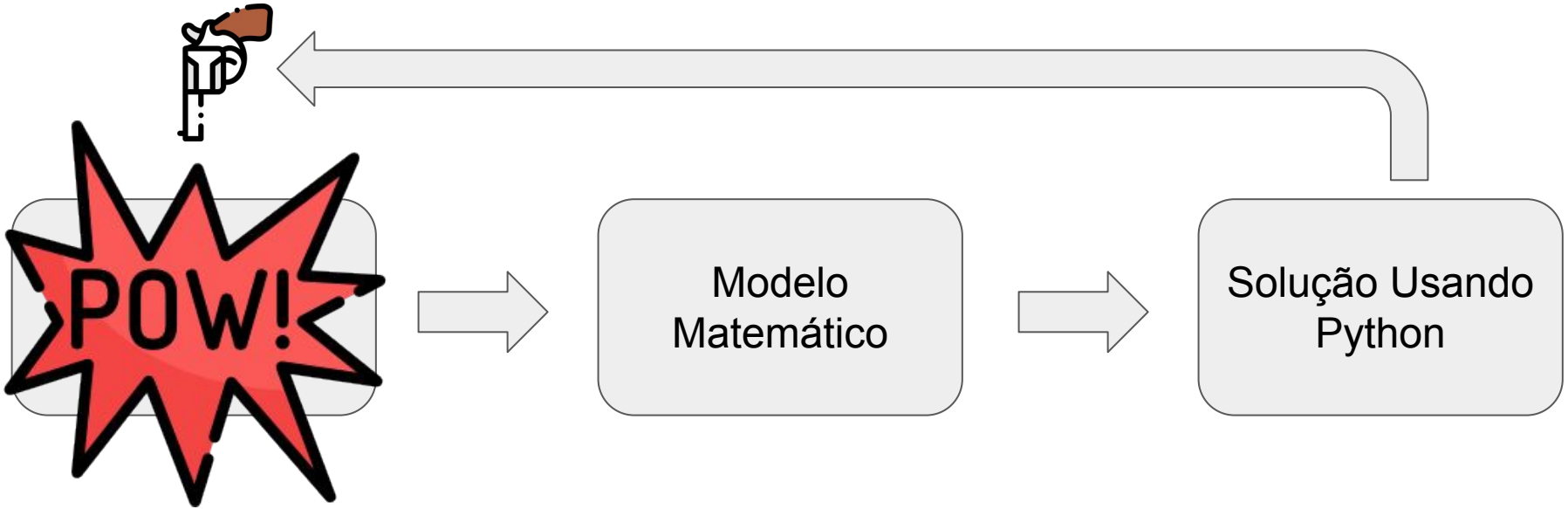

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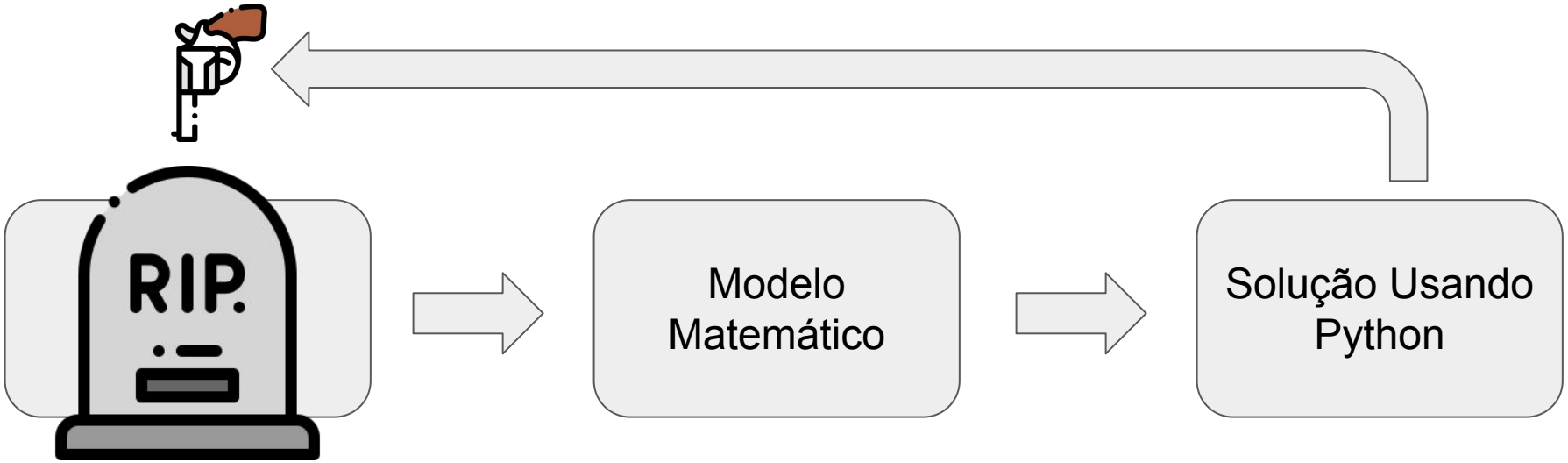
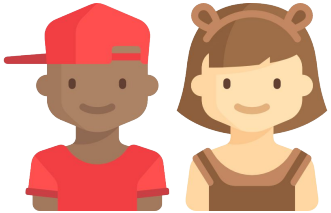
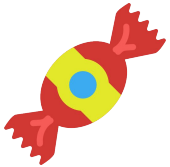
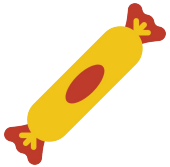
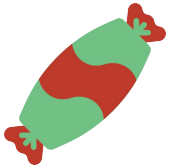
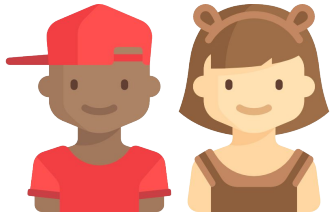




Você e seu irmão ganharam, de sua avó, uma caixa com 6 bombons diferentes.





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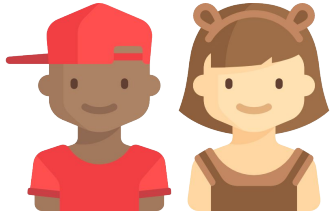

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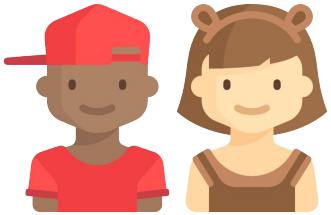
$$\begin{aligned} & \min \ f(x) \\ s.a. \quad & g(x) \geq b \\ & x \geq 0 \end{aligned}$$



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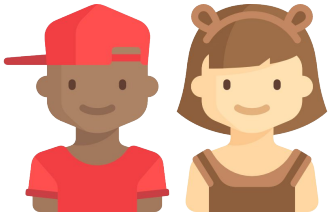

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Função Objetivo

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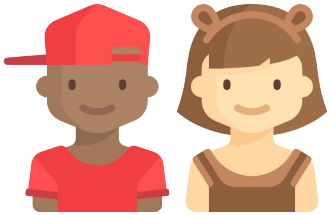

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Função Objetivo

Restrições

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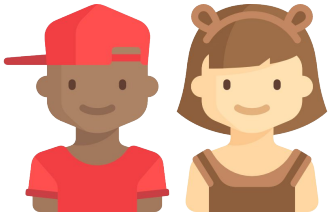

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Função Objetivo

Restrições

Variáveis de Decisão

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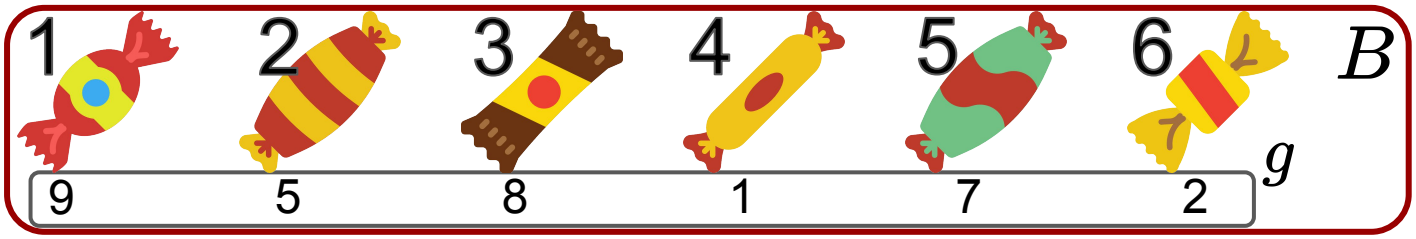
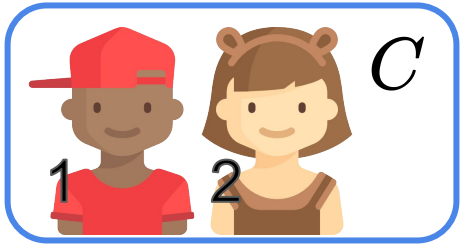
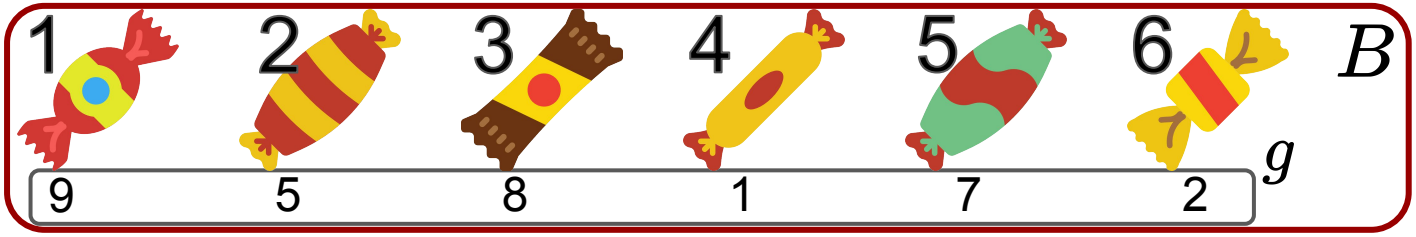
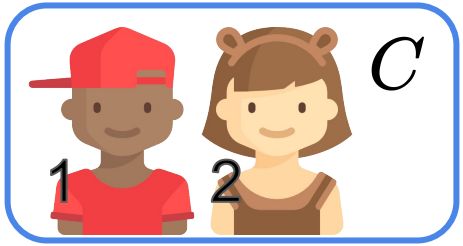




Variáveis de decisão





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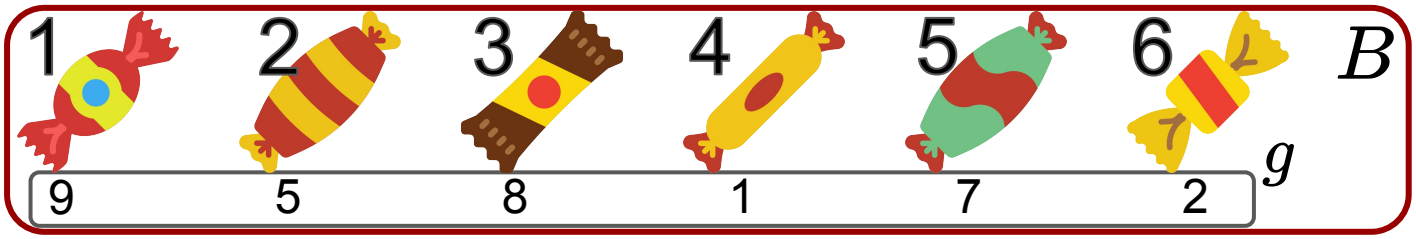

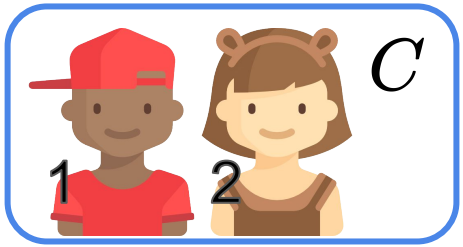

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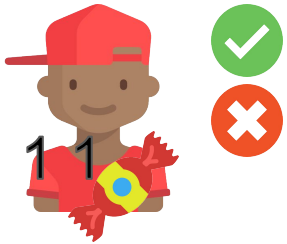
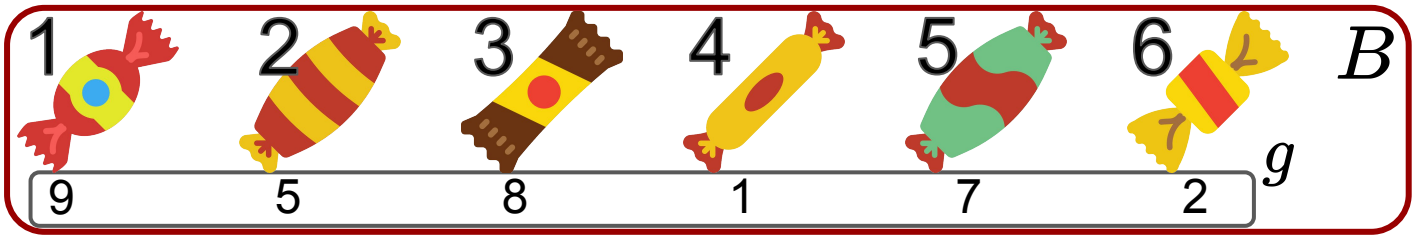
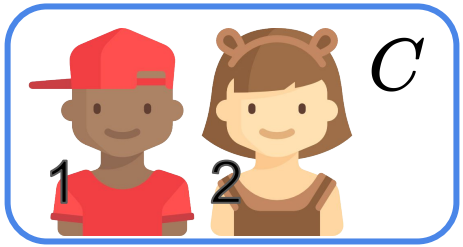
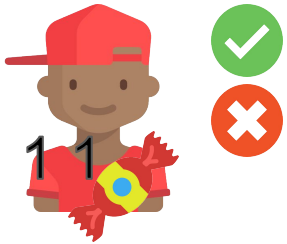
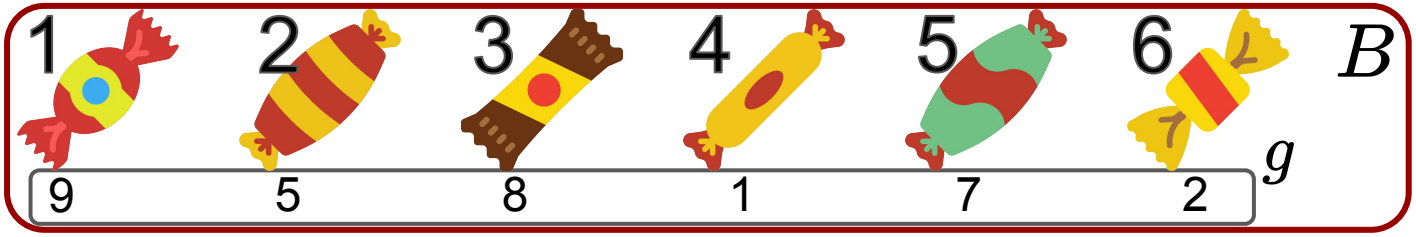


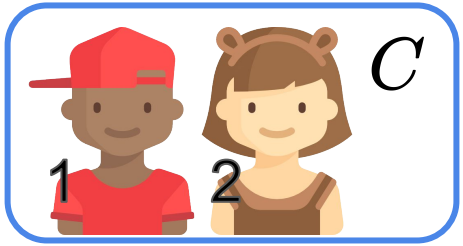




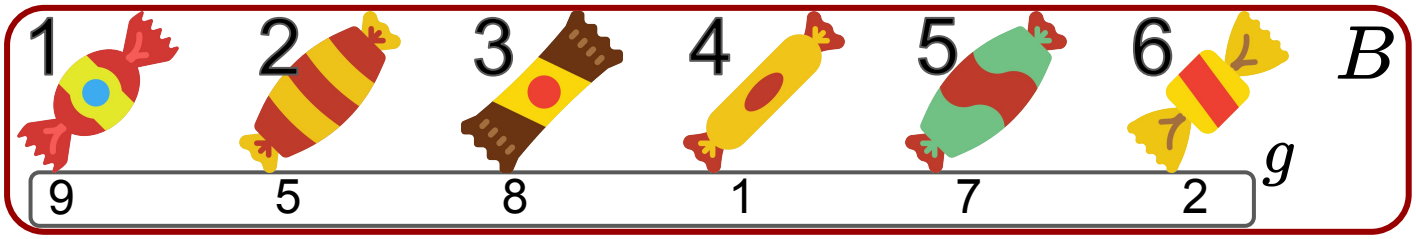
Variáveis de decisão

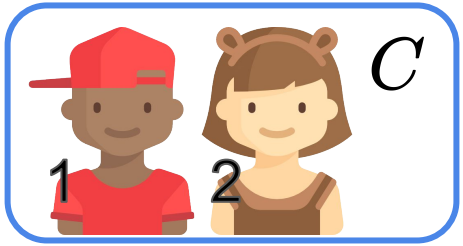






Variáveis de decisão

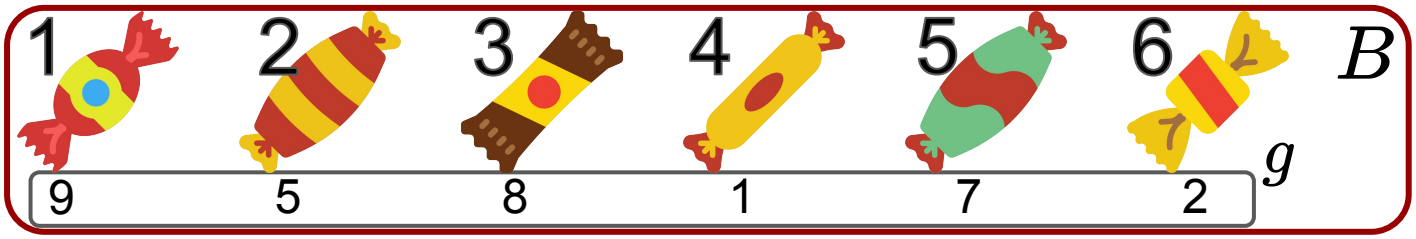


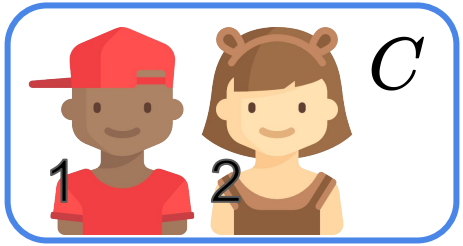


Variáveis de decisão

$$x_{11} \quad x_{21} \quad x_{31} \quad x_{41} \quad x_{51} \quad x_{61}$$

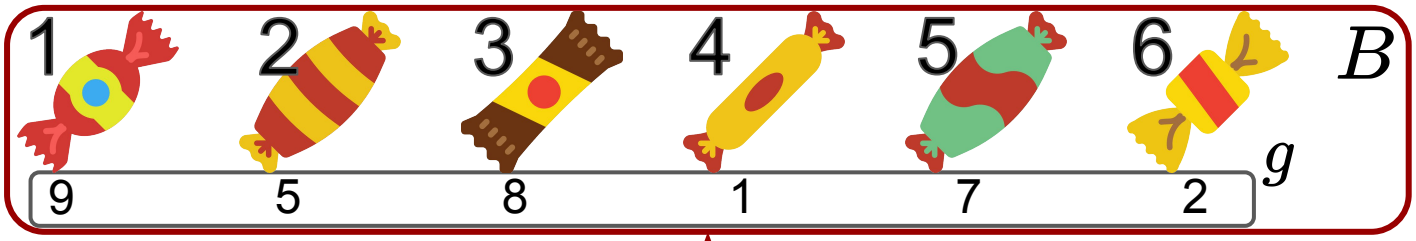
$$x_{12} \quad x_{22} \quad x_{32} \quad x_{42} \quad x_{52} \quad x_{62}$$

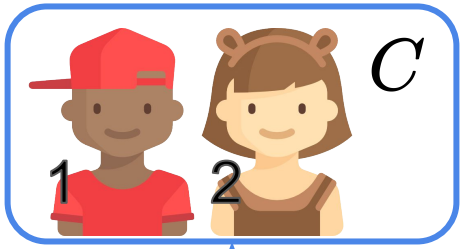




Variáveis de decisão

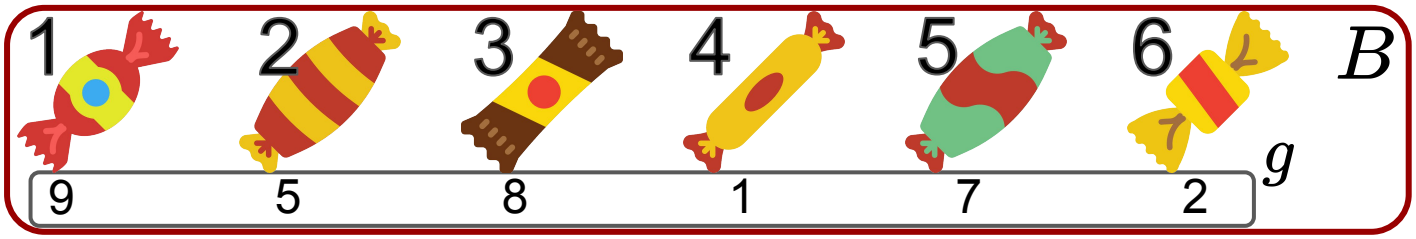
$$\begin{matrix} x_{11} & x_{21} & x_{31} & x_{41} & x_{51} & x_{61} \\ x_{12} & x_{22} & x_{32} & x_{42} & x_{52} & x_{62} \end{matrix} = 0 \; ou \; 1$$

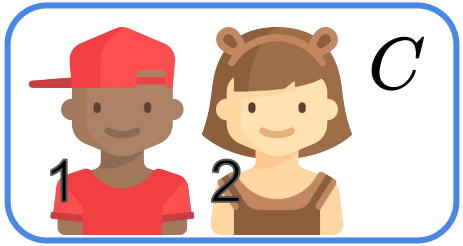




Variáveis de decisão

$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$





Modelo Matemático

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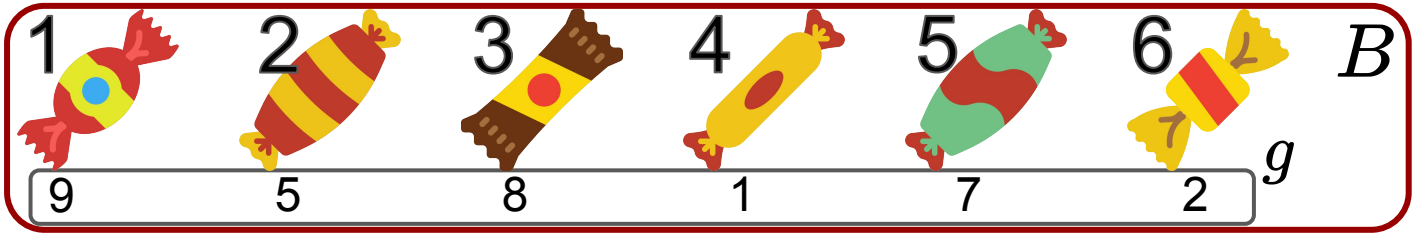
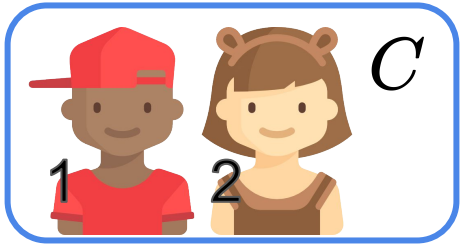
- Função objetivo
- Restrições
- Variáveis de decisão





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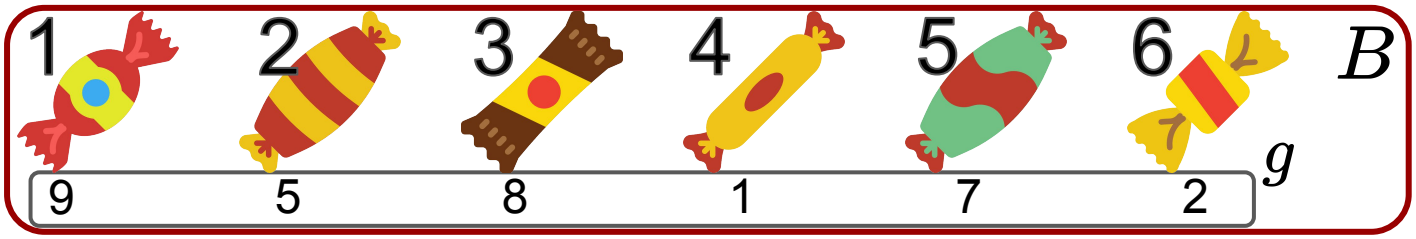

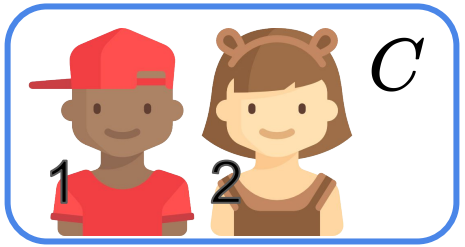

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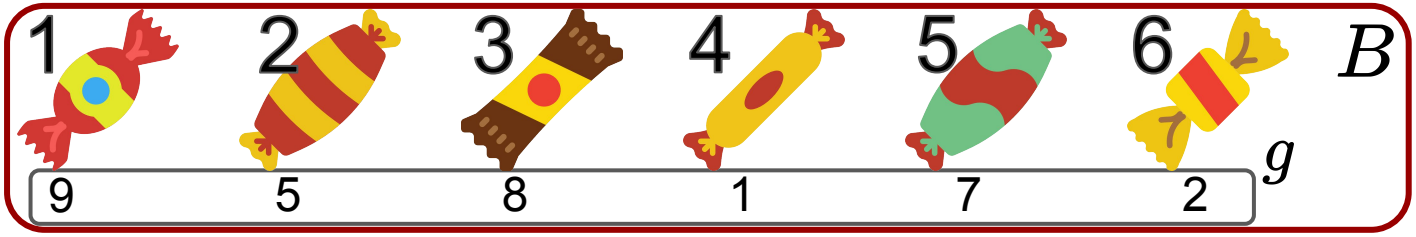
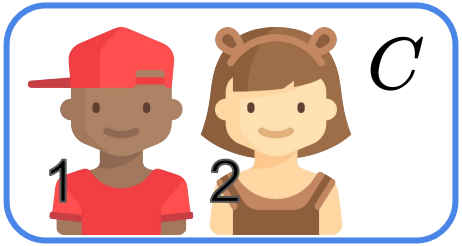
$$\min\Big(\quad \text{  } \quad - \quad \text{  } \quad \Big)$$





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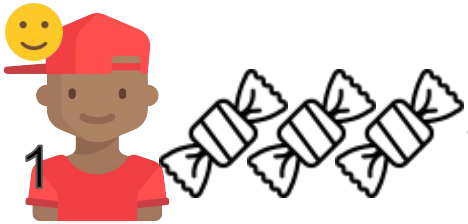
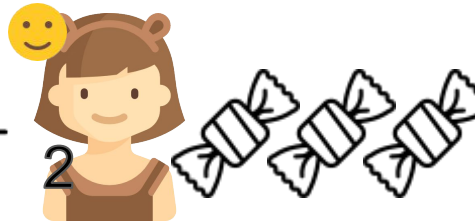
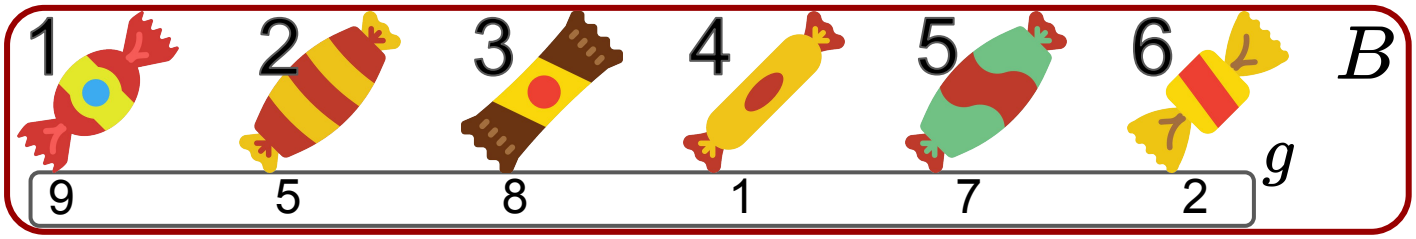
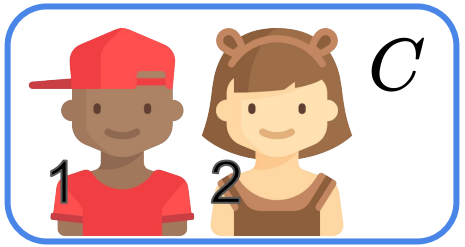
$$\min($$  $$-$$  $$)$$





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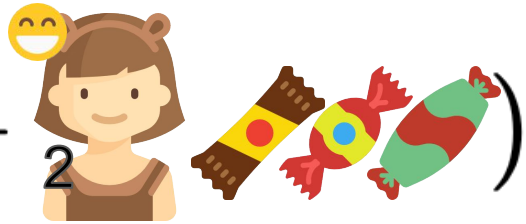
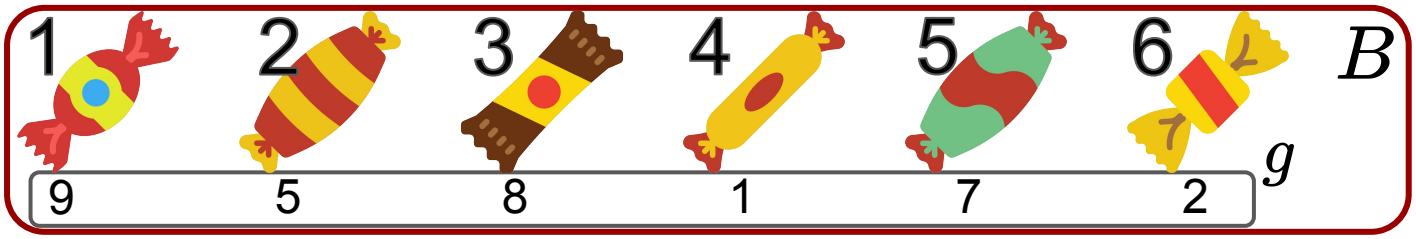
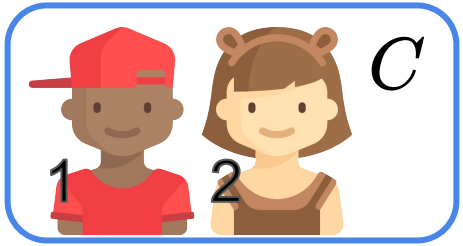
$$\min($$  $$-$$  $$)$$

5 + 1 + 2

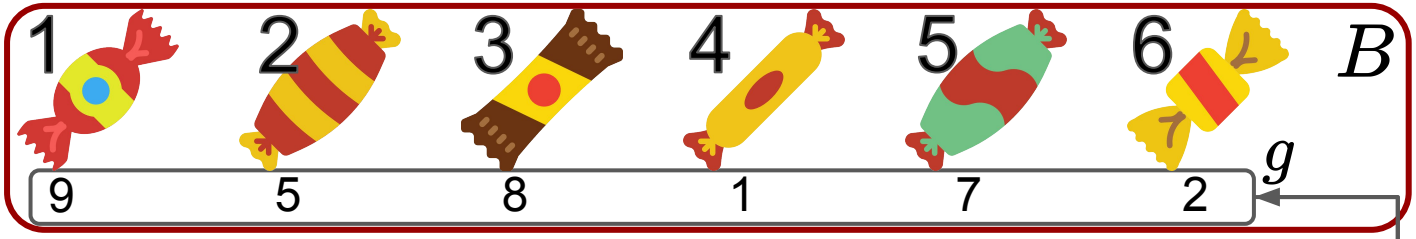
8 + 9 + 7

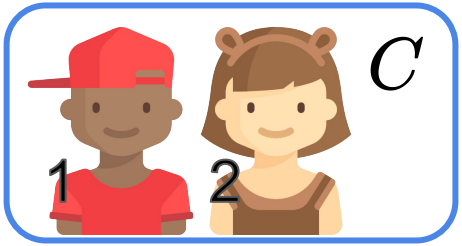




Função Objetivo

$$\min\left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)$$





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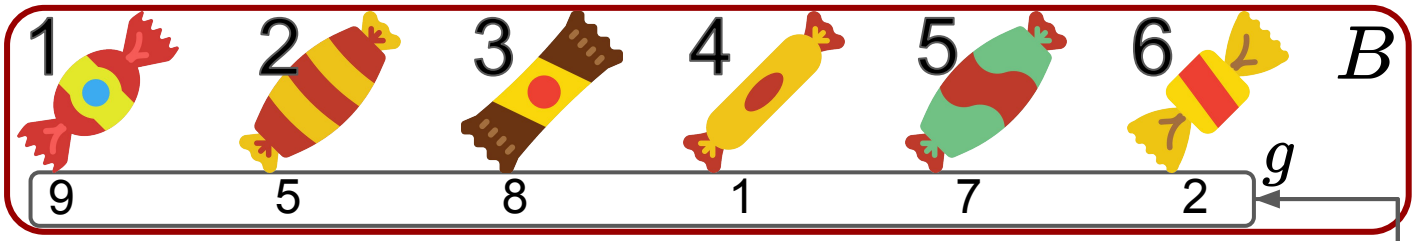
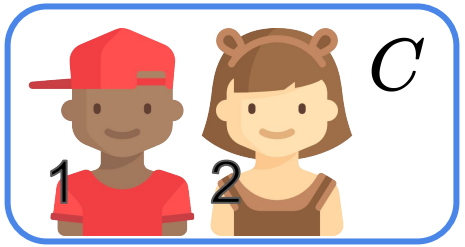
$$\min\left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)$$

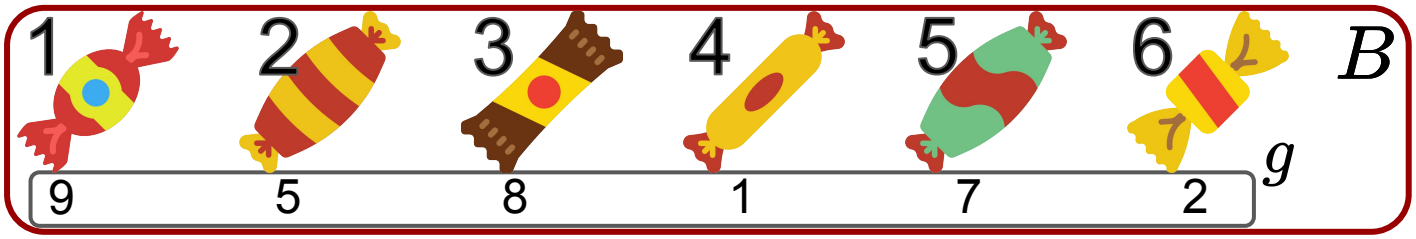


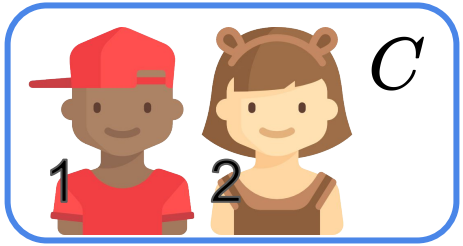


Função Objetivo

$$\min\left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)$$

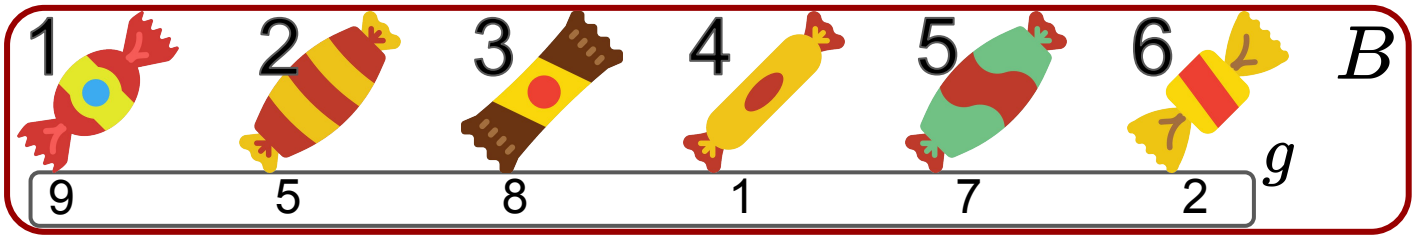
Variáveis de decisão

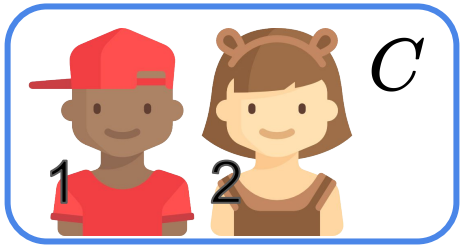




Função Objetivo

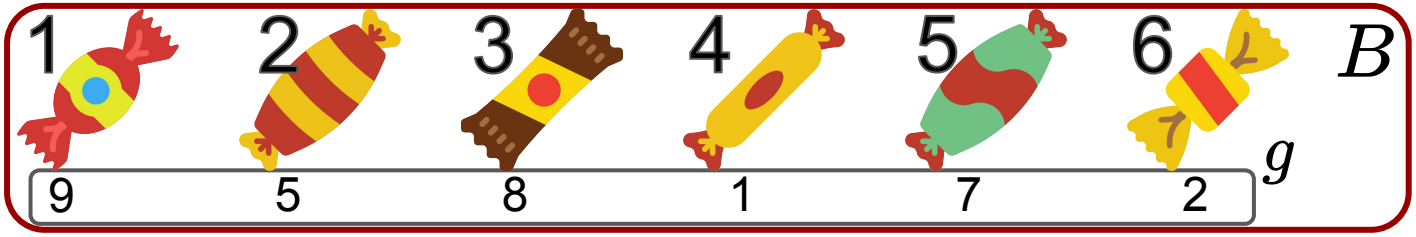
$$\min\left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)$$

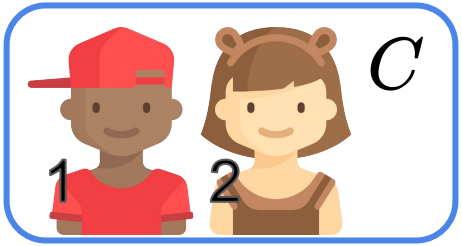




Função Objetivo

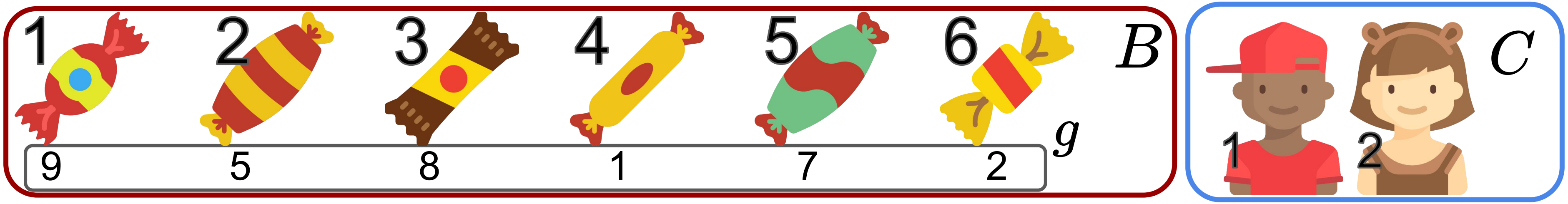
$$\min\left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)$$



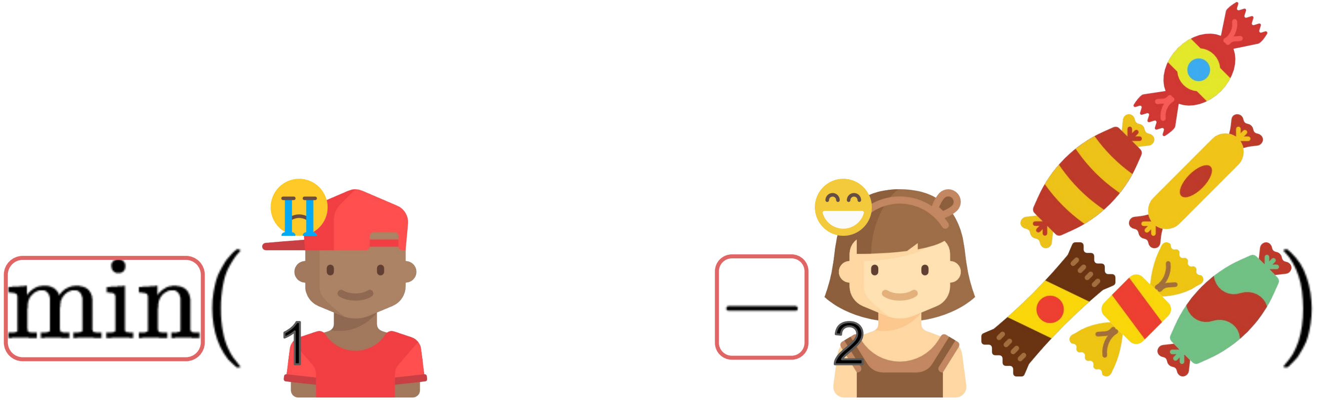


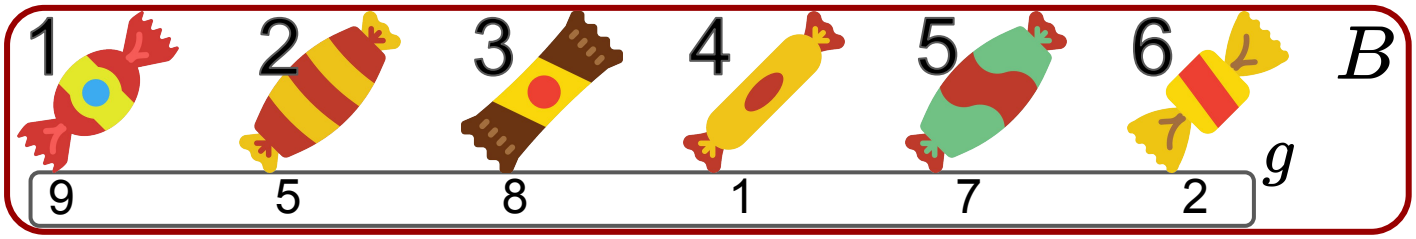
Função Objetivo

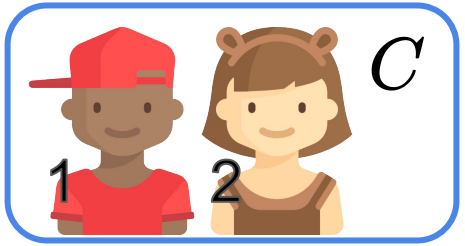
$$\min\left(\quad - \quad \right)$$



Função Objetivo

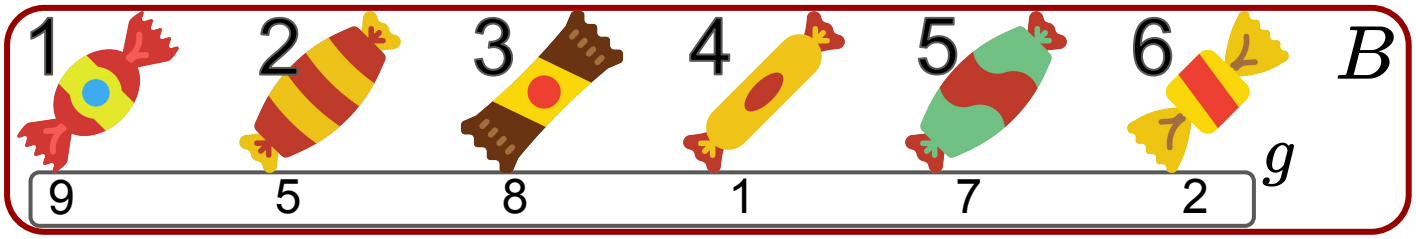


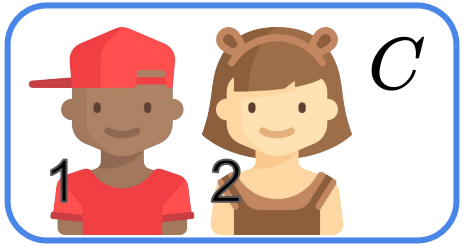




Função Objetivo

$$\min\left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)^2$$



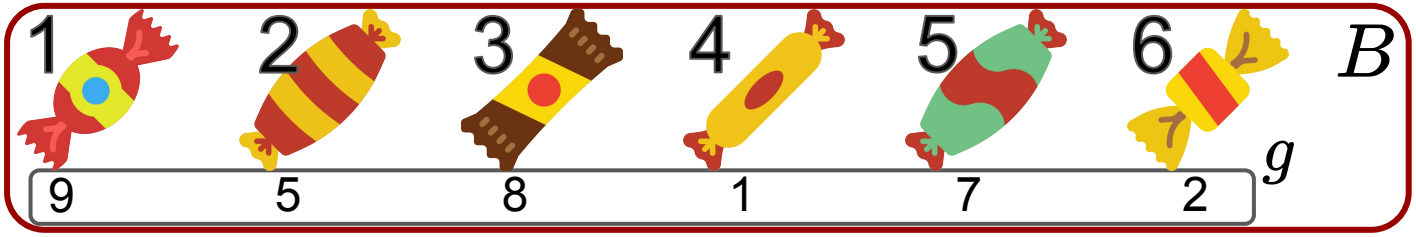


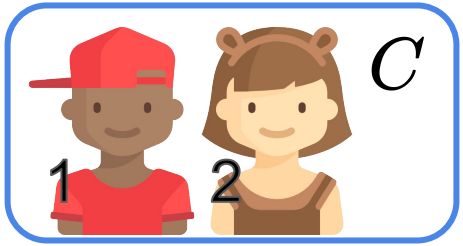
Modelo Matemático

$$\min\left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)^2$$ → Função objetivo

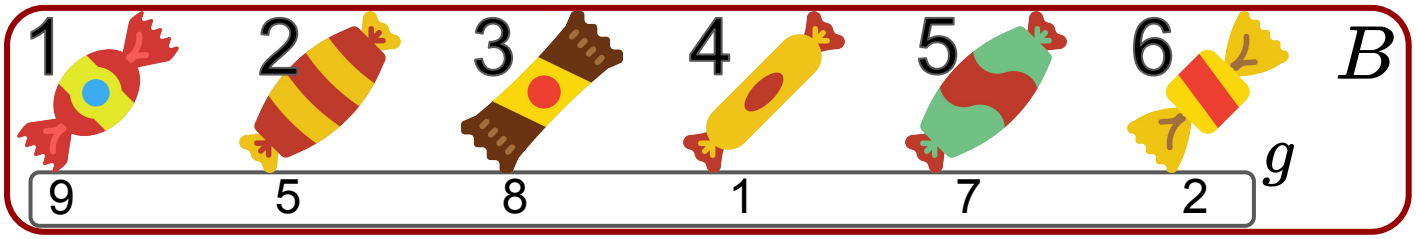
Restrições

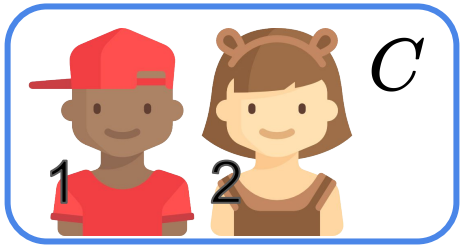
$$x_{ij} = \{0,1\}, \forall i \in B \wedge \forall j \in C$$ → Variáveis de decisão





Restrições

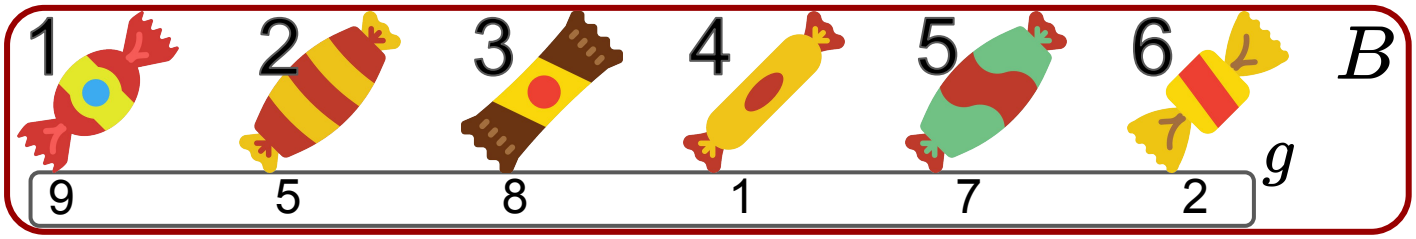


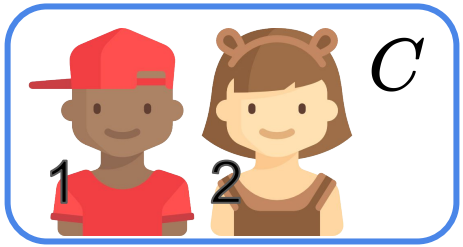


Restrições

$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$

$$\begin{matrix} x_{11} & x_{21} & x_{31} & x_{41} & x_{51} & x_{61} \\ x_{12} & x_{22} & x_{32} & x_{42} & x_{52} & x_{62} \end{matrix}$$



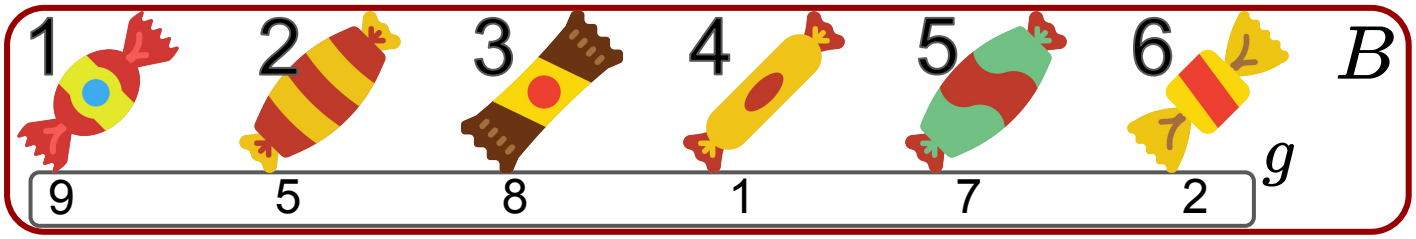


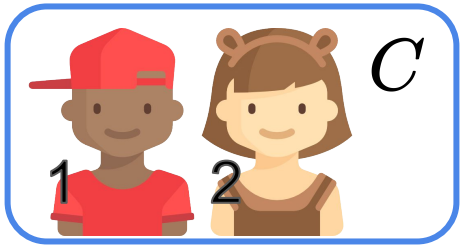
Restrições

$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$

$$x_{11} = 0 \; x_{21} = 0 \; x_{31} = 0 \; x_{41} = 0 \; x_{51} = 0 \; x_{61} = 0$$

$$x_{12} = 0 \; x_{22} = 0 \; x_{32} = 0 \; x_{42} = 0 \; x_{52} = 0 \; x_{62} = 0$$





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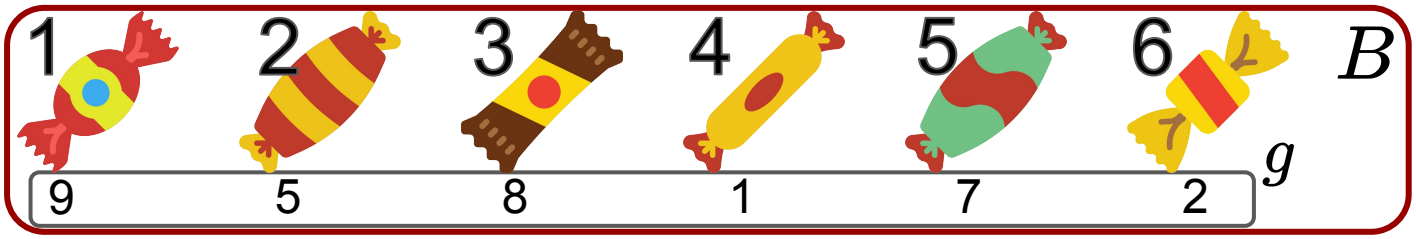
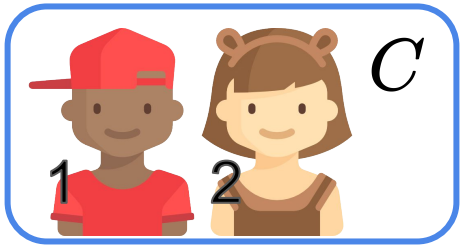


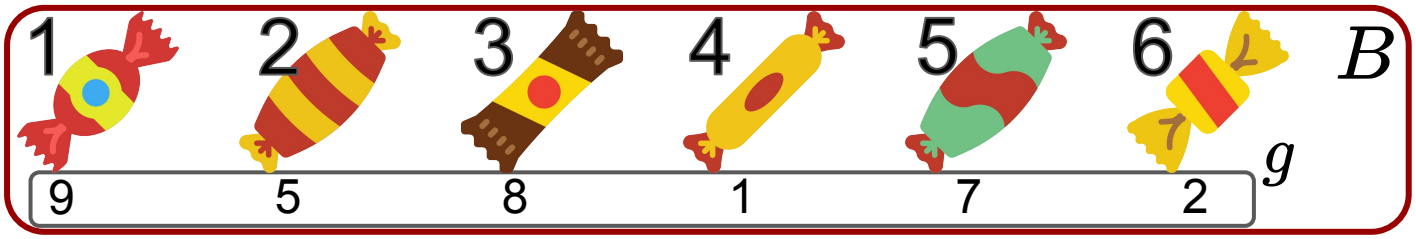


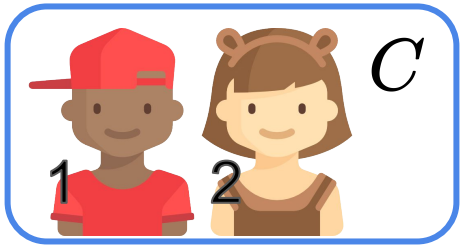
Restrições

$$x_{ij} = \{0, 1\}, \forall i \in B \land \forall j \in C$$

$$x_{11} = 1 \; x_{21} = 1 \; x_{31} = 1 \; x_{41} = 1 \; x_{51} = 1 \; x_{61} = 1$$

$$x_{12} = 1 \; x_{22} = 1 \; x_{32} = 1 \; x_{42} = 1 \; x_{52} = 1 \; x_{62} = 1$$





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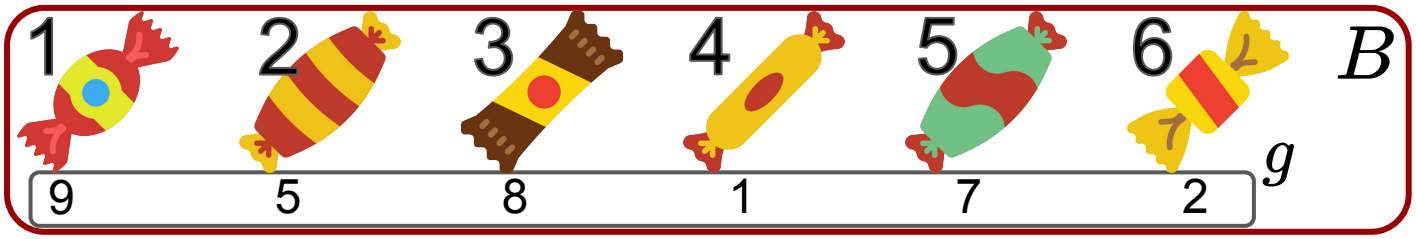

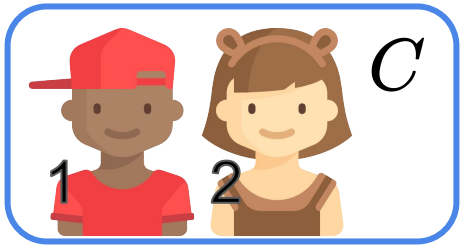

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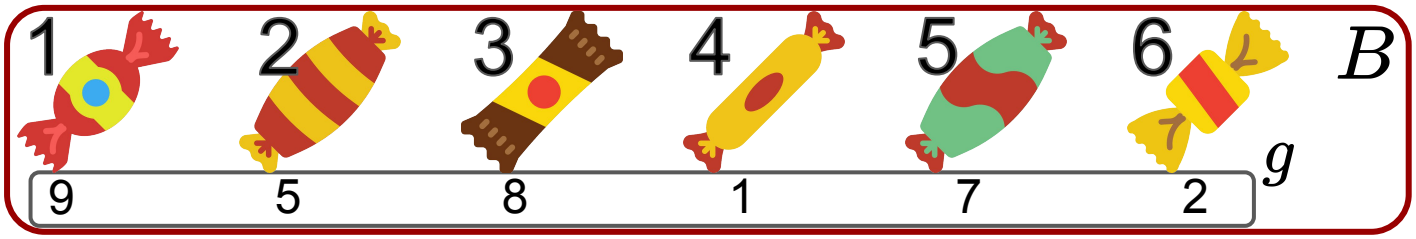

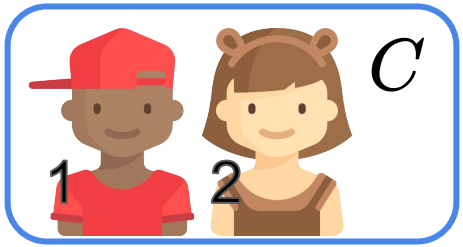

Restrições

$$x_{11} = 0\ x_{21} = 0\ x_{31} = 0\ x_{41} = 0\ x_{51} = 0\ x_{61} = 0$$
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$$x_{11} = 1\ x_{21} = 0\ x_{31} = 1\ x_{41} = 0\ x_{51} = 0\ x_{61} = 1$$
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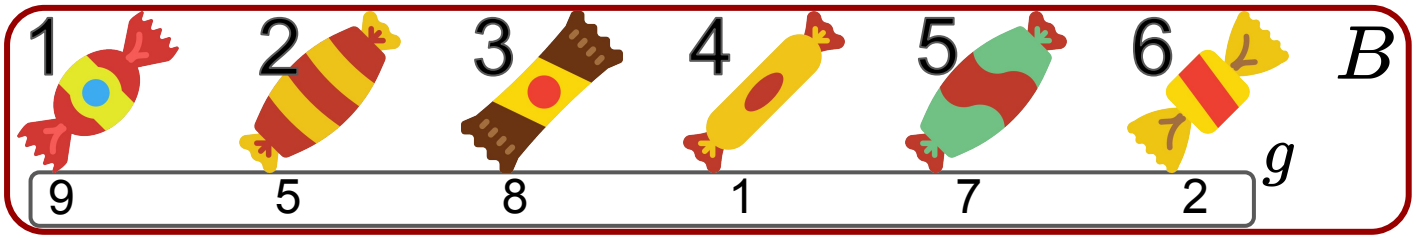
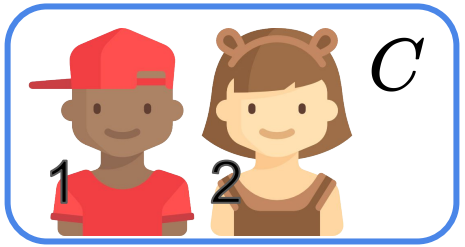
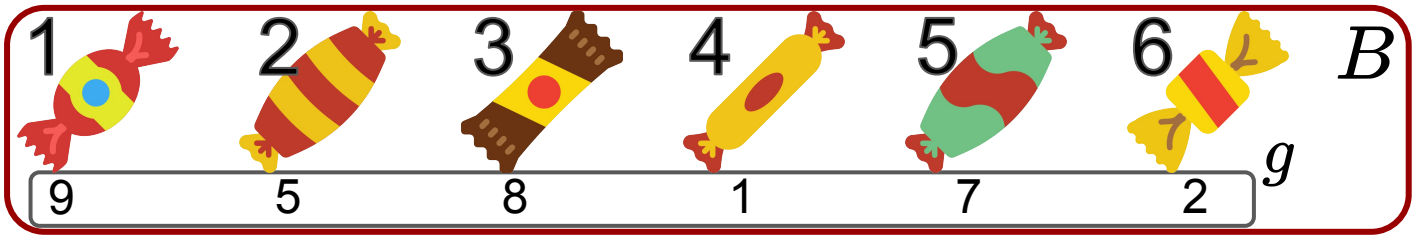
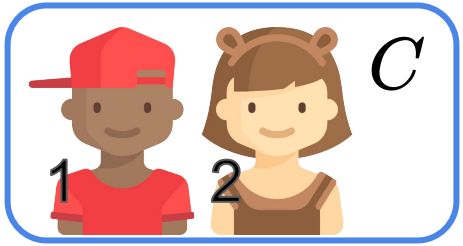


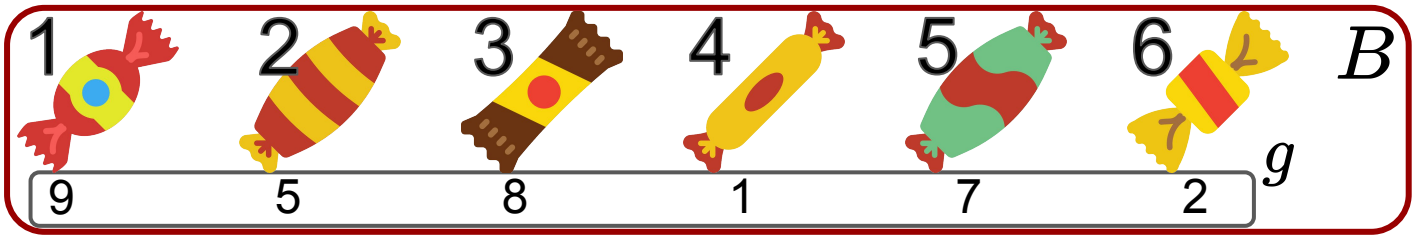


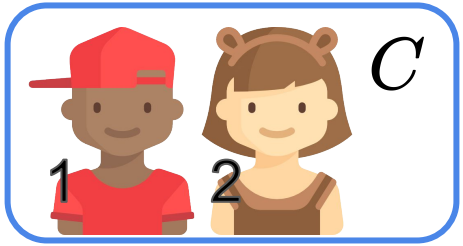
$$
\begin{aligned}
x_{11} + x_{12} &= 1 \\
x_{21} + x_{22} &= 1 \\
x_{31} + x_{32} &= 1 \\
x_{41} + x_{42} &= 1 \\
x_{51} + x_{52} &= 1 \\
x_{61} + x_{62} &= 1
\end{aligned}
$$





$$x_{i1} + x_{i2} = 1, \forall i \in B$$



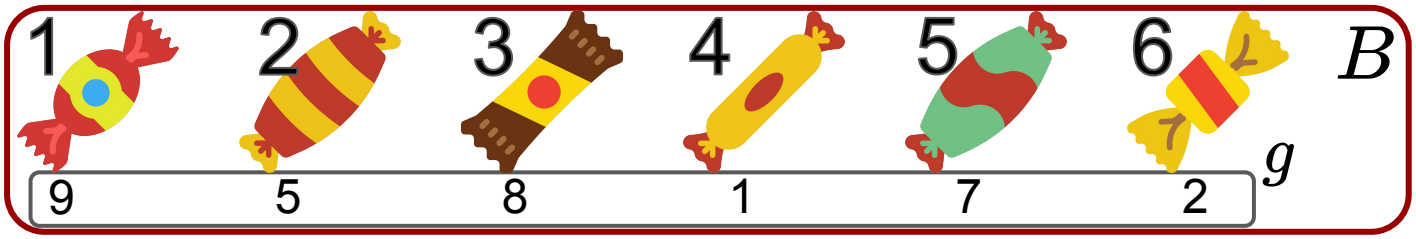


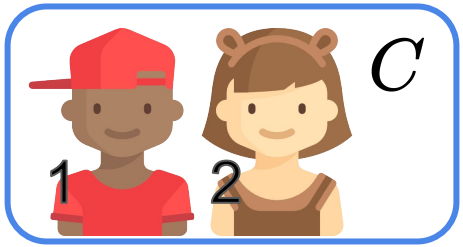
Modelo Matemático

$$\min\left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)^2 \quad \rightarrow \text{Função objetivo}$$

$$x_{i1} + x_{i2} = 1, \forall i \in B \quad \rightarrow \text{Restrições}$$

$$x_{ij} = \{0,1\}, \forall i \in B \wedge \forall j \in C \quad \rightarrow \text{Variáveis de decisão}$$

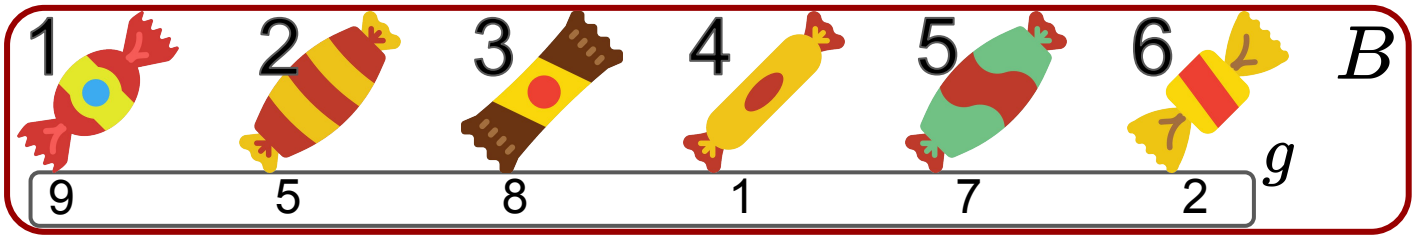


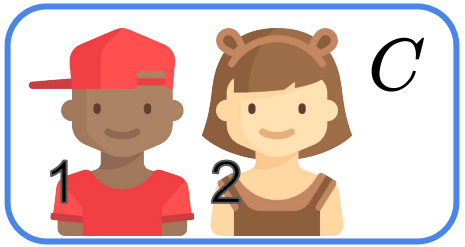


Você e seu irmão ganharam uma caixa com 6 bombons da sua avó.

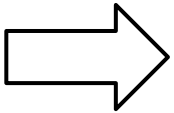
Qual a melhor forma de dividi-los, fazendo com que vocês dois fiquem igualmente contentes com o resultado da divisão?

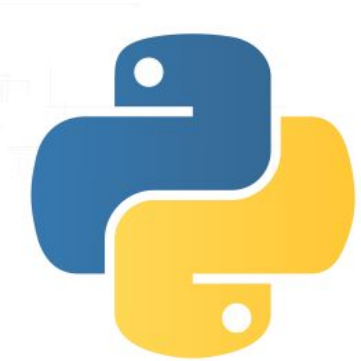
$$\begin{aligned} \min \quad & \left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)^2 \\ s.t. \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\ & x_{ij} = \{0,1\}, \forall i \in B \wedge \forall j \in C \end{aligned}$$

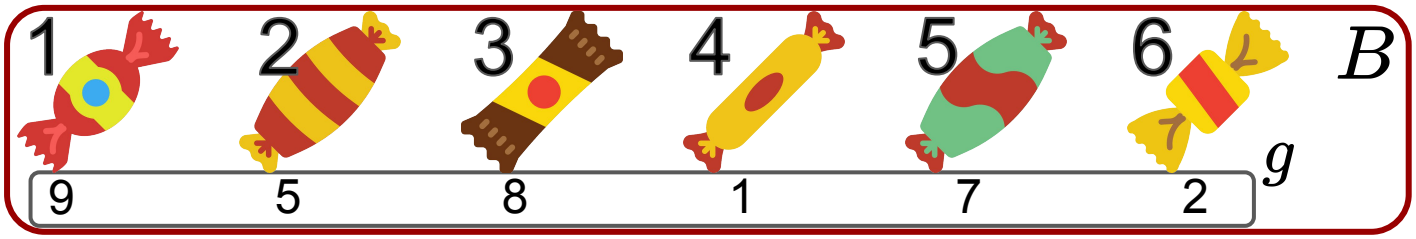


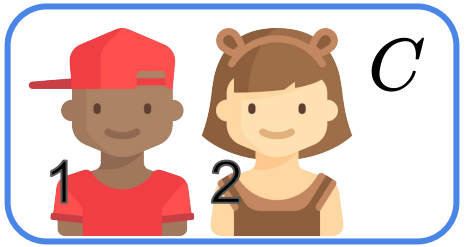


$$
\begin{aligned}
\min \quad & \left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)^2 \\
s.t. \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\
& x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C
\end{aligned}
$$

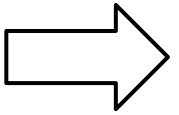








$$
\begin{aligned}
\min \quad & \left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)^2 \\
s.t. \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\
& x_{ij} = \{0,1\}, \forall i \in B \wedge \forall j \in C
\end{aligned}
$$



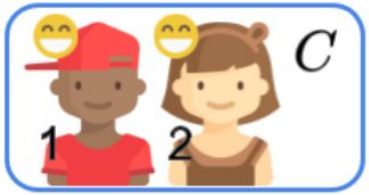
Modelo Matemático

$$
\begin{aligned}
\min \quad & \left(\sum_B g_i x_{i1} - \sum_B g_i x_{i2}\right)^2 \\
s.t. \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\
& x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C
\end{aligned}
$$

Modelo Matemático

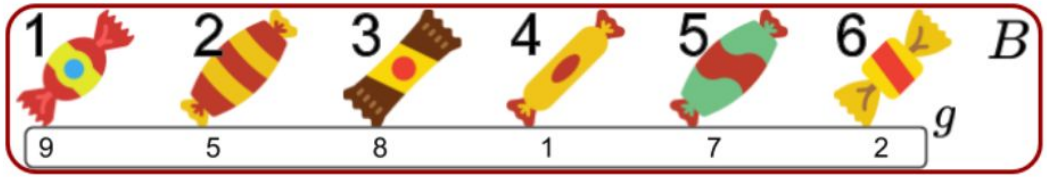
$$\begin{aligned} \min \quad & (\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2 \\ s.t. \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\ & x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C \end{aligned}$$

In [1]:
```
import pyomo.environ as poe
modelo = poe.AbstractModel()
```



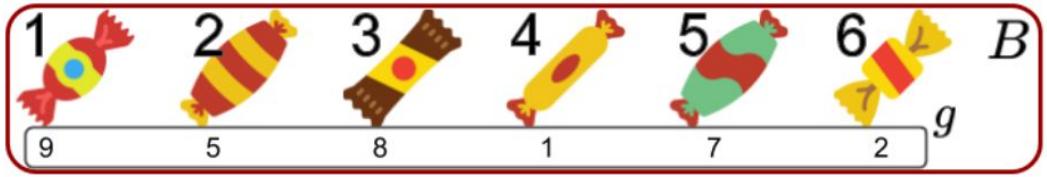
Conjunto representando as crianças:

In [2]:
```
modelo.criancas = poe.RangeSet(1,2)
```



Conjunto representando os bombons:

In [3]:
```
modelo.bombons = poe.Set()
```



Conjunto representando os bombons:

In [3]:
```
modelo.bombons = poe.Set()
```

Nivel de gostosura dos bombons:

In [4]:
```
modelo.gostosura = poe.Param(modelo.bombons, within=poe.NonNegativeReals)
```

Variáveis de decisão:

$$x_{11}\ x_{21}\ x_{31}\ x_{41}\ x_{51}\ x_{61}$$
$$x_{12}\ x_{22}\ x_{32}\ x_{42}\ x_{52}\ x_{62}$$

- 1, se o bombom i for para a criança j
- 0, caso contrário

In [5]:
```
modelo.x = poe.Var(modelo.bombons, modelo.criancas, within=poe.Binary)
```

Função objetivo:

$$\min \quad (\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2$$

In [6]:

```
def funcao_objetivo(modelo):
    return((sum(modelo.gostosura[i] * modelo.x[i, 1] for i in modelo.bombons))
           - sum(modelo.gostosura[i] * modelo.x[i, 2] for i in modelo.bombons))
**2

modelo.OBJ = poe.Objective(rule=funcao_objetivo)
```

Restrições:

$$s.t. \quad x_{i1} + x_{i2} = 1, \forall i \in B$$

In [7]:
```
def funcao_restricao_de_cobertura(modelo, i):
    return modelo.x[i, 1] + modelo.x[i, 2] == 1

modelo.restricao_de_cobertura = \
    poe.Constraint(modelo.bombons, rule=funcao_restricao_de_cobertura)
```

In [8]:
```
! cat candy_box_problem_instance.dat
```

```
# AMPL format

set bombons := 1 2 3 4 5 6;

param gostosura :=
1 9
2 5
3 8
4 1
5 7
6 2
;
```

In [9]:
```
!pyomo solve candy_box_problem.py candy_box_problem_instance.dat --solver=bonmin
```

```
[    0.00] Setting up Pyomo environment
[    0.00] Applying Pyomo preprocessing actions
[    0.00] Creating model
[    0.01] Applying solver
[    0.05] Processing results
    Number of solutions: 1
    Solution Information
      Gap: None
      Status: optimal
      Function Value: 0.0
    Solver results file: results.yml
[    0.05] Applying Pyomo postprocessing actions
[    0.05] Pyomo Finished
```

In [10]:
```
!grep -B 1 -A 99 "#   Solution Information" results.yml
```

```
# ------------------------------------------------------------
#   Solution Information
# ------------------------------------------------------------
Solution:
- number of solutions: 1
  number of solutions displayed: 1
- Gap: None
  Status: optimal
  Message: bonmin\x3a Optimal
  Objective:
    OBJ:
      Value: 0
  Variable:
    x[1,2]:
      Value: 1
    x[2,2]:
      Value: 1
    x[3,1]:
      Value: 1
    x[4,1]:
      Value: 1
    x[5,1]:
      Value: 1
    x[6,2]:
      Value: 1
  Constraint: No values
```

8

1

7

9

5

2

16

16

Problema de particionamento de número

- Divisão de bombons

Problema de particionamento de número

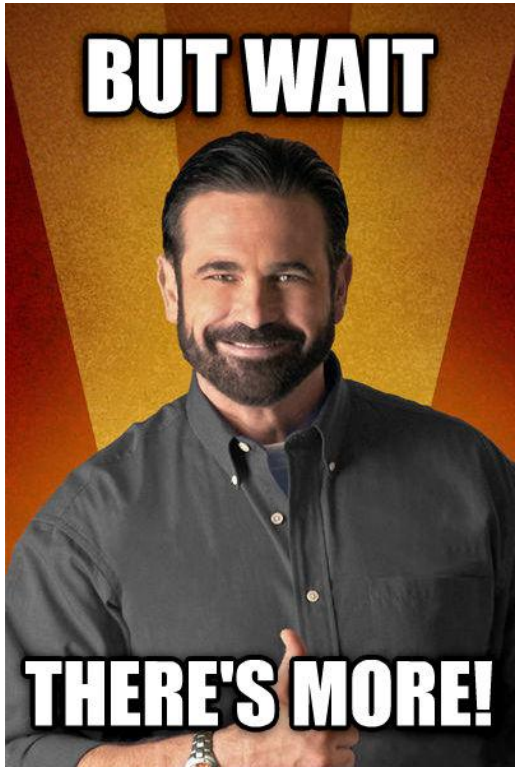
- Divisão de bombons
- Divisão de tarefas domésticas

Problema de particionamento de número

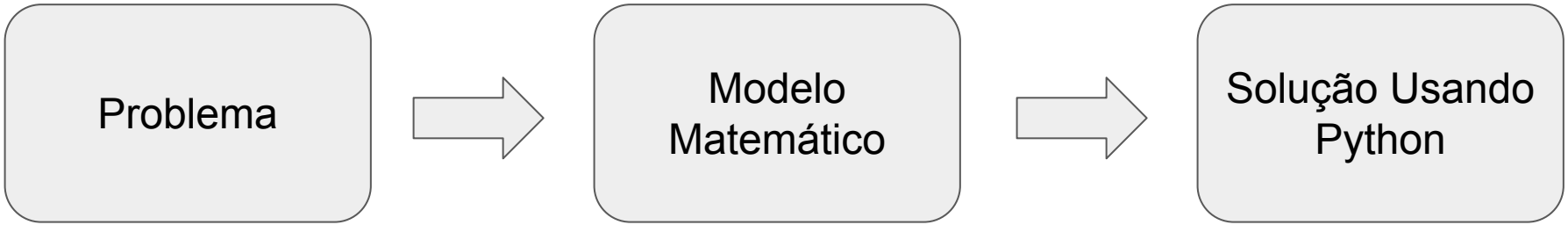
- Divisão de bombons
- Divisão de tarefas domésticas
- Divisão de tarefas entre processadores

Problema de particionamento de número

- Divisão de bombons
- Divisão de tarefas domésticas
- Divisão de tarefas entre processadores
- Divisão de linhas entre empresas de ônibus


BUT WAIT
THERE'S MORE!

3 simples passos



- Alocação de viagens a ônibus
- Alocação de viagens a motoristas
- Escala semanal
- Planejamento de viagem

Quer saber mais?

Hart, William E., Carl Laird, Jean-Paul Watson, David L. Woodruff, Gabriel A. Hackebeil, Bethany L. Nicholson, and John D. Siirola. **Pyomo – Optimization Modeling in Python**. Springer, 2017.

WPLEX. **Artigos – Pesquisa e Desenvolvimento para Mobilidade Urbana**. Disponível em: <https://wplex.com.br/artigos.html>

https://github.com/renan-eccel/candy-box-problem

Muito obrigado pela oportunidade!

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